Entanglement Transitions in Random Qudit Circuits

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Introduction

- **Motivation :** Closed quantum systems display a thermal entanglement entropy scaling proportional to the system volume. However, a bath coupling can constrain this entropy growth to the system's boundaries alone. This phenomenon, numerically realized in unitary circuits with projective measurements, has shown to allow a fine-tuned control over the thermal entanglement entropy, in various many-body systems composed of qubits.

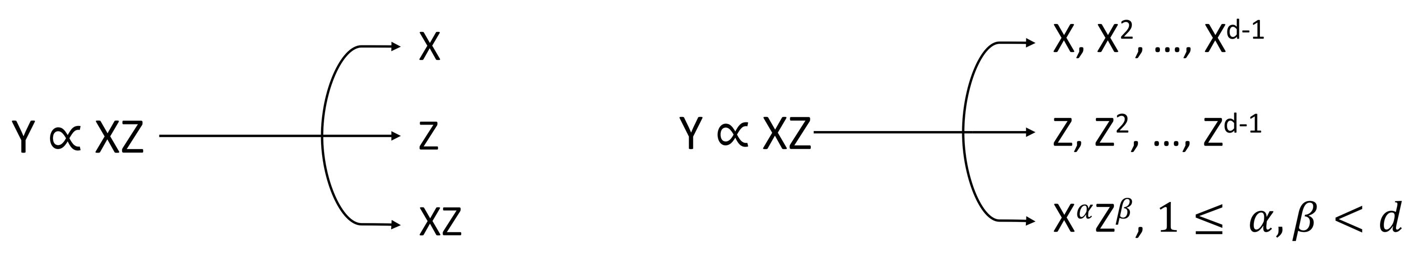
- **Question :** How does the entanglement transition from volume-law to area-law thermal states alter in many-body systems with higher local Hilbert-space dimensions, i.e. in many-body qudit systems?

- **Approach :** We study random quantum circuits composed of Clifford unitaries and local-projective measurements, which exhibit evolution in the absence of symmetries.

Qudit Arithmetic

What fundamentally changes while considering n-qudit unitaries?

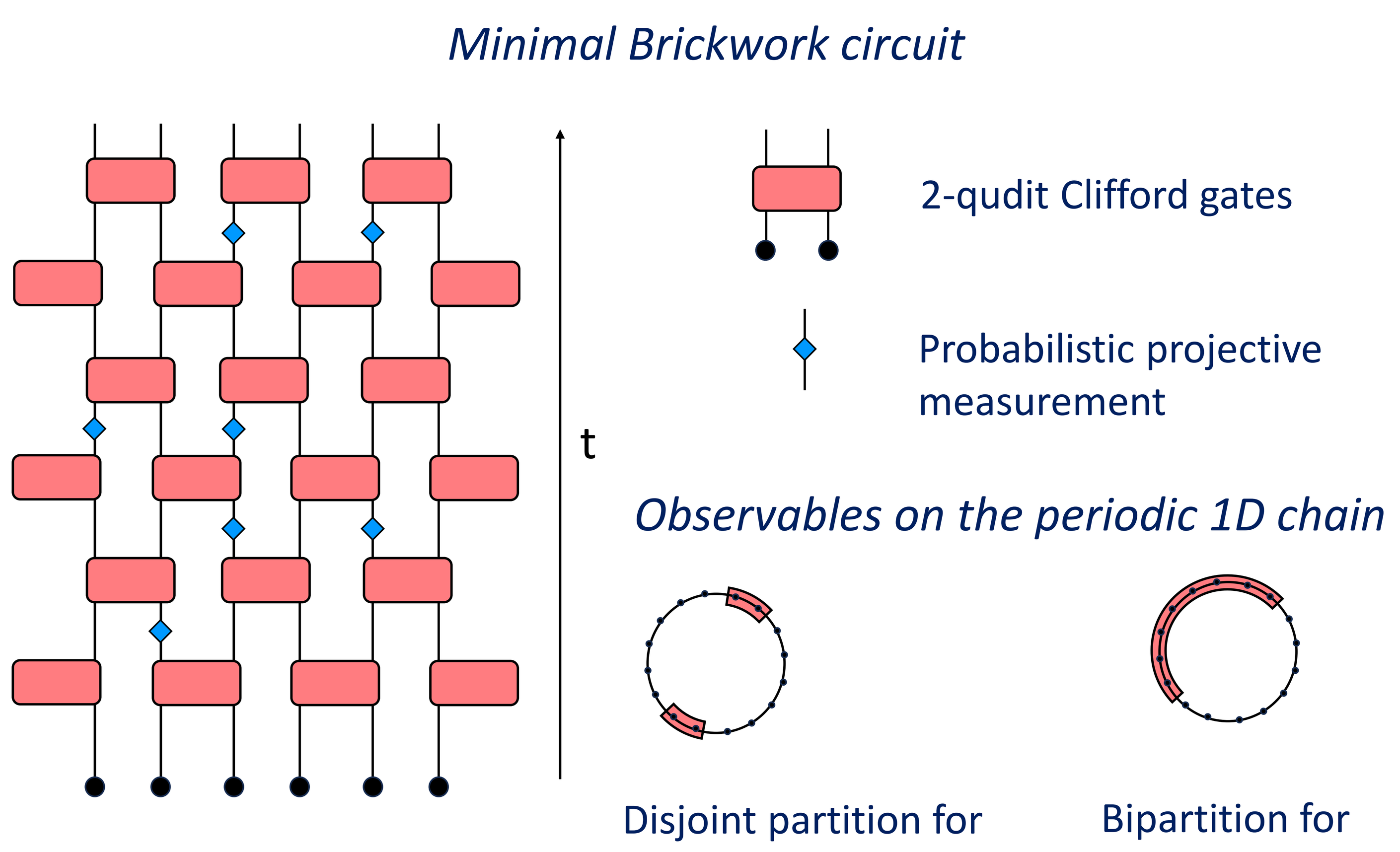
- *Clifford Group $Cl(2,n)$* : Set of unitaries that map the n-qubit Pauli group to itself.
- *Clifford Group $Cl(d,n)$* : Set of unitaries that map the n-qudit Weyl-Heisenberg group to itself.



1-qubit map **1-qudit map**

- An n-qudit state prepared and measured in the computational basis, undergoing Clifford evolution, can be **efficiently simulated classically**, using $2n * (log_2(d-1)+1)+(d-1)$ bits.
- The Clifford group $Cl(d,n)$ also forms a **unitary 2-design** over the Haar measure.

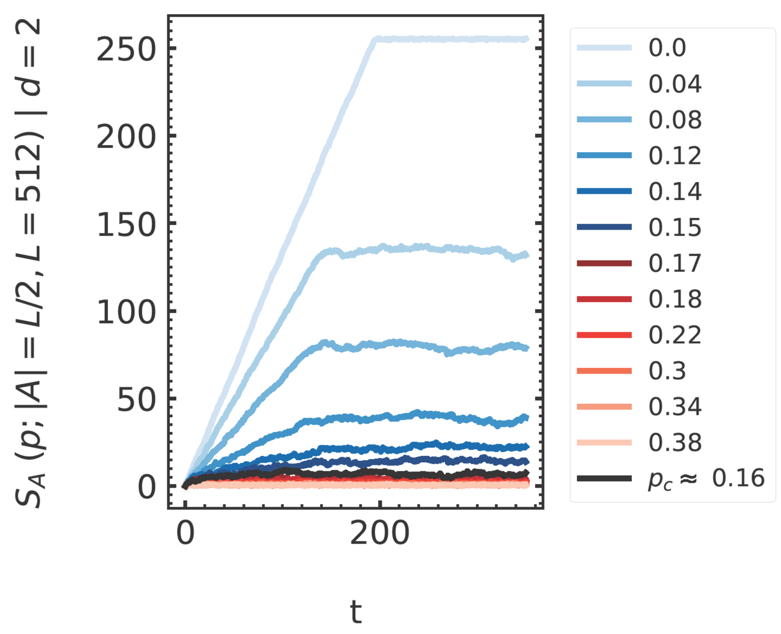
Circuit Model



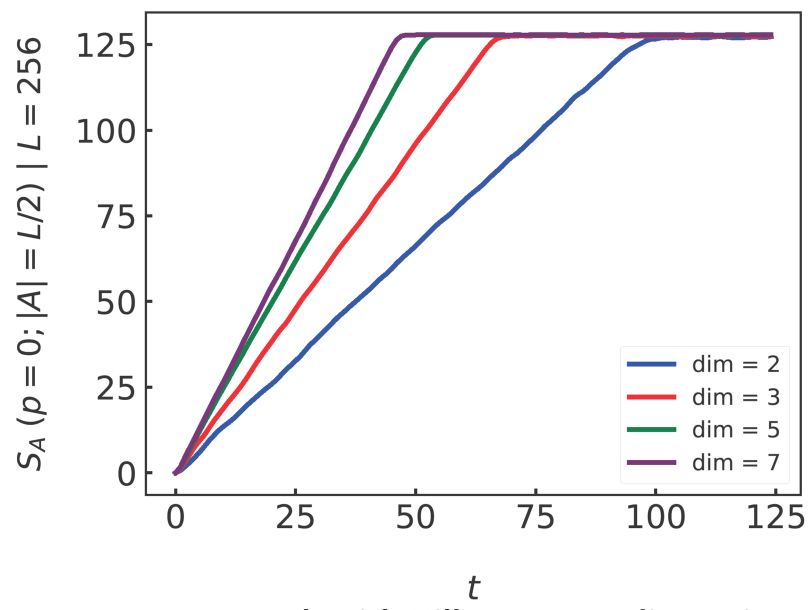
Disjoint partition for mutual information

Bipartition for Rényi entropy

Thermalization Dynamics



Bipartite entropy dynamics with varying measurement probabilities



v_E growth with Hilbert-space dimension

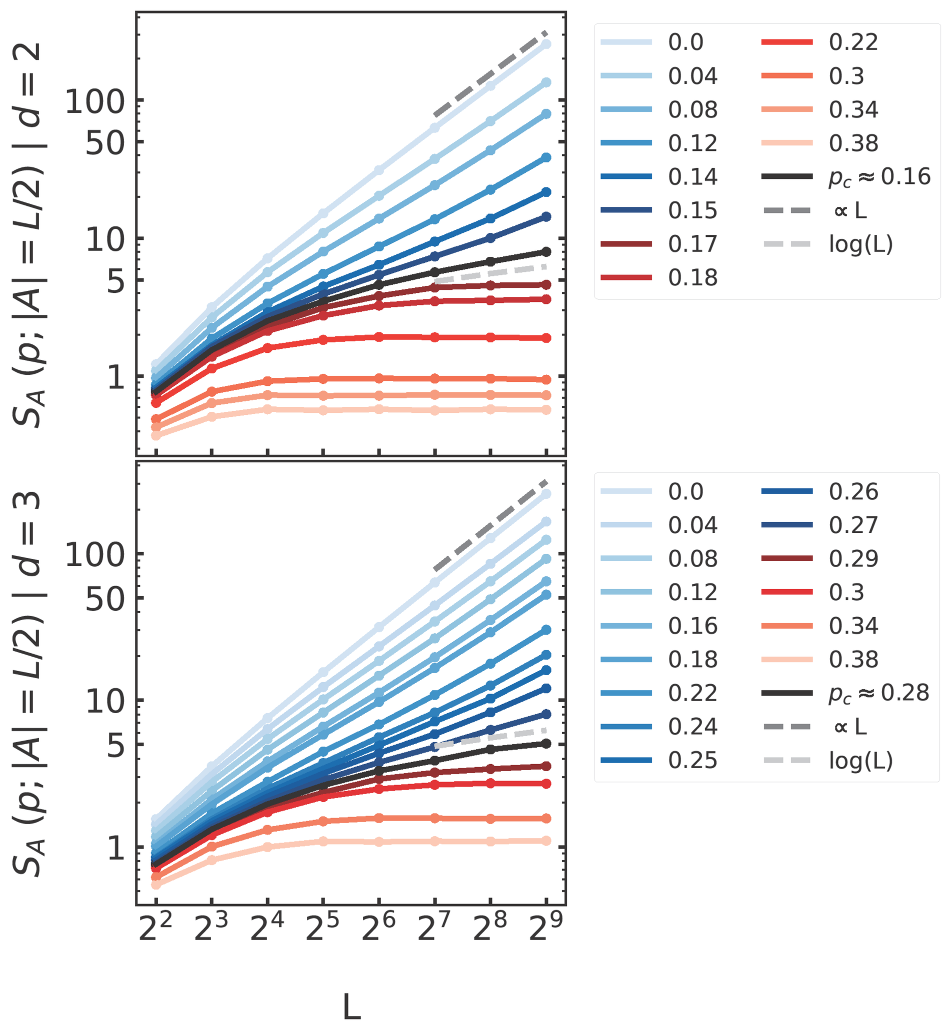
- Increasing measurement probabilities diminish the thermal entanglement entropy. This entanglement growth can be modelled linearly with subleading corrections.

$$< S(t) > = v_E\, t + B t^{\frac{1}{3}}$$

- Increasing the local Hilbert-space dimension also shows a convergent growth in v_E.

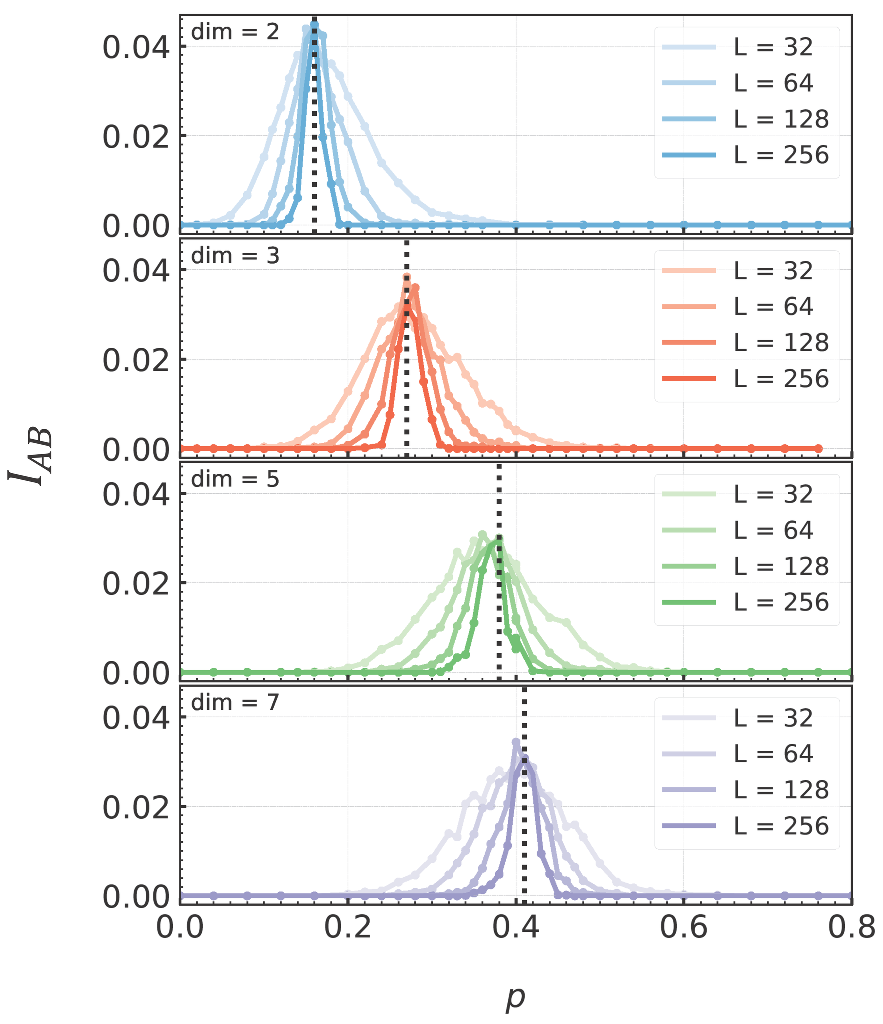
Thermal Properties

Criticality and Scaling



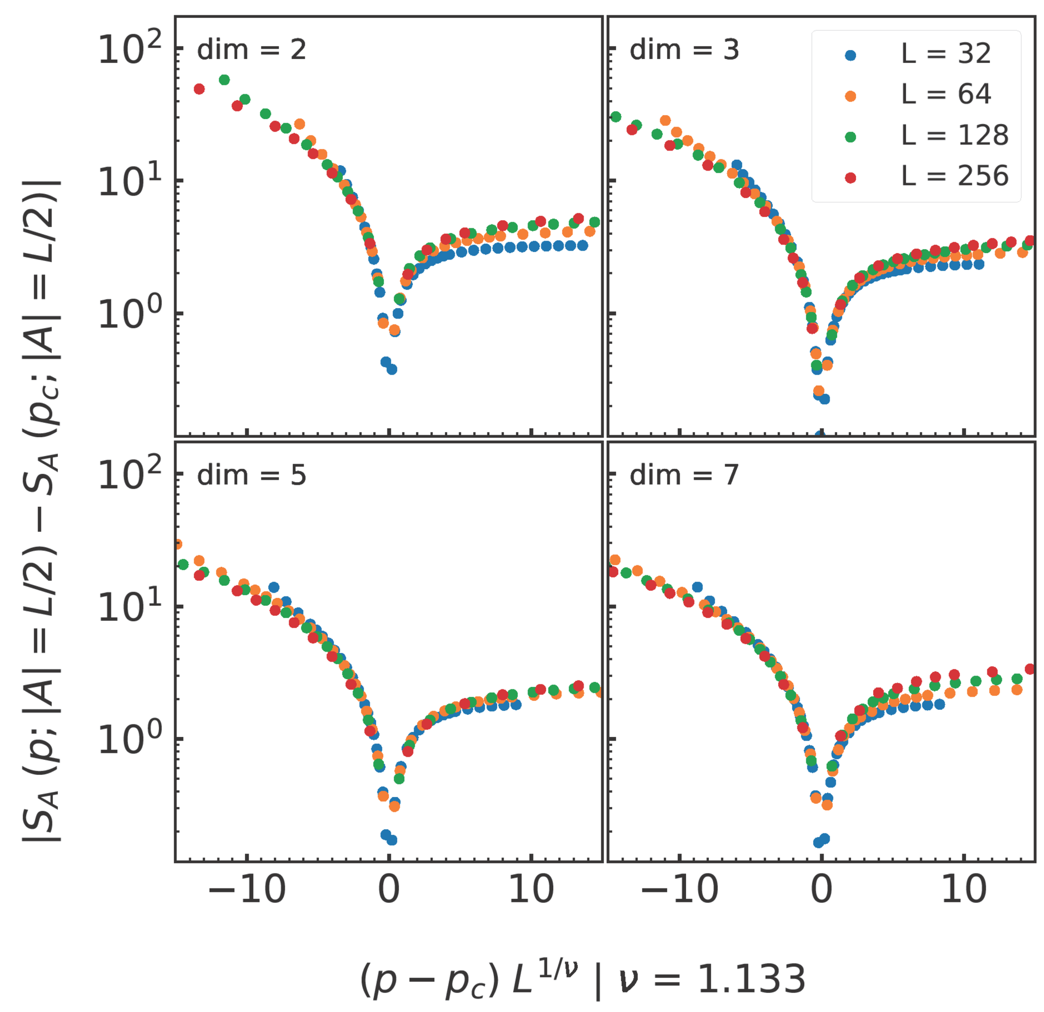
Thermal entropy scaling with system size

Convergence of Criticality



Convergence of critical measurement probability

Universality



Entropy collapse comparing exponents

Akin to qubit circuits, the thermal entropy undergoes logarithmic scaling at the critical probability of measurement.

However, mutual information indicates a convergent shift in the phase transition with increasing with the local Hilbert-space dimension.

Preliminary scaling collapses indicates that the phase transitions share critical exponents across local Hilbert-space dimensions.

OUTLOOK

Entanglement transitions in random qudit circuits have shown to preserve the nature of the phase transition, from a volume-law phase to an area-law phase through a logarithmically-scaling critical point. However, the critical point and the speed of entropy-spread v_E show a convergent increase, which naturally leads us to ask:

1. Can a ***classical model*** *be formulated* in that captures the entanglement transition in the large Hilbert-space dimension limit?
2. Furthermore, can the projective measurements be altered to realize ***distinct area-law phases***?

References

1. Li et al. *Physical Review B, 100(13), 134306 (2019).*
2. Heinrich, M. *Doctoral dissertation, Universität zu Köln* (2021).
3. Nahum, A. et al. *Physical Review X, 7(3), 031016 (2017)*

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